ON A CASE OF INTEGRABLE EQUATIONS OF PERTURBED MOTION OF A SATELLITE

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PMM Vol.29, № 4, 1965, p. 751

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(Received October 3, 1964)

The paper considers the case of satellite motion under the influence of the Newtonian force and an additional perturbing force perpendicular to the velocity and lying in the plane of the trajectory. The equations of motion are integrated by quadratures if the magnitude F of the force depends only on the velocity v and the distance r of the satellite from the center of attraction.

The equations of plane motion of the point M of unit mass (satellite) in polar coordinates are

$$r^{\cdot\cdot} - r\varphi^{\cdot 2} + kr^{-2} = F_r, \qquad r\varphi^{\cdot\cdot} + 2r^{\cdot}\varphi^{\cdot} = F_\varphi \qquad (k = \text{const}) \tag{1}$$

The dot indicates differentiation with respect to time; F_r, F_φ are the projections of the perturbing force F along the directions r and φ. For the assumptions made about the force we have

$$F_r = -F(r, v)\frac{r\varphi^{\cdot}}{\sqrt{r^2\varphi^{\cdot 2} + r^{\cdot 2}}}, \qquad F_\varphi = F(r, v)\frac{r^{\cdot}}{\sqrt{r^2\varphi^{\cdot 2} + r^{\cdot 2}}} \tag{2}$$

We seek the solution of the system (1), (2) subject to the initial conditions

$$r = r_0, \quad r^{\cdot} = r_0^{\cdot}, \quad \varphi = \varphi_0, \quad \varphi^{\cdot} = \varphi_0^{\cdot}, \quad t = t_0 \tag{3}$$

If the square of the velocity v is differentiated with respect to time

$$v^2 = r^2\varphi^{\cdot 2} + r^{\cdot 2} \tag{4}$$

then, on the strength of (1) and (2), there results an integrable equation

$$\frac{dv^2}{dt} = -\frac{2k}{r}\frac{dr}{dt} \tag{5}$$

From (5) we find the energy integral

$$v^2 - 2kr^{-1} = C_1, \qquad C_1 = r_0^2\varphi_0^{\cdot 2} + r_0^{\cdot 2} - 2kr_0^{-1} \tag{6}$$

The second equation in (1) can be expressed as

$$(r^2\varphi^{\cdot})^{\cdot} = F(r, v)\, rv^{-1}r^{\cdot} \tag{7}$$

It integrates as

$$r^2\varphi^{\cdot} = \Phi(r, C_1, C_2) \tag{8}$$

$$\Phi(r, C_1, C_2) = \int_{r_0}^{r} F(r, v)\frac{r}{v}\,dr + C_1, \quad C_1 = r_0^2\varphi_0^{\cdot}, \quad v = \frac{1}{C_1 + 2kr^{-1}} \tag{9}$$

Eliminating the derivative $\varphi^{\cdot}$ from (6) we obtain the integrable equation

$$r^{\cdot} = (2kr^{-1} + C_1 - \Phi^2(r, C_1, C_2))^{1/2} \tag{10}$$

This yields the quadrature

$$t - t_0 = \int_{r_0}^{r} (2kr^{-1} + C_1 - \Phi^2(r, C_1, C_2))^{-1/2}\,dr \tag{11}$$

From (8) and (10) we find the relationship between r and φ as

$$\varphi - \varphi_0 = \int_{r_0}^{r} \Phi r^{-2}(2kr^{-1} + C_1 - \Phi^2)^{-1/2}\,dr \tag{12}$$

In the general case the motion of the point M occurs in a ring which is bounded by the circles with radii r_1, r_2. The numbers r_1, r_2 are simple roots of the radicand in (10). The case $r_2 = \infty$ is possible.

A.I. Lur'e has indicated to the author that certain cases of electron motion in the electromagnetic fields [1] lead to the equations of the form (1) and (2).

BIBLIOGRAPHY

1. Boguslavskii, S.A., Puti elektronov v elektromagnitnykh poliakh (Paths of the electrons in the electromagnetic fields). Mosgublit, 1929.

Translated by V.A.C.